# Analytical model

This section will outline and clarify the analytical models in a step-by-step manner for two situations: (1) when there is only a single node with single and batch arrival, and each packet consumes the same amount of energy, and (2) when there are three interconnected nodes with single and batch arrival in the network, and each packet consumes the same amount of energy.

## Scenario 1

In this section, we will focus on a scenario where packets come with single or batch arrival can be divided into two priorities: high priority (HP) and low priority (LP), and both types of packets require the same amount of energy. Within each priority level, the packets are serviced in a first-come, first-served (FCFS) order. Once a packet enters the queue, it cannot be preempted, which means that an HP packet can always overtake an LP packet, but once an LP packet is in service, it cannot be interrupted. Additionally, there is a chance that a packet waiting in the queue may leave the system due to impatience. It is worth noting that when a packet is ready to be serviced, it first checks if there is enough energy in the energy queue. If there isn't enough energy, the packet may use a regular battery with a given probability. The model diagram, state balance equations, iterative algorithm, and performance metrics can be found below.

### Model diagram

Fig. 3 - 1 illustrates the components of the model used in scenario 1, which include a finite packet queue, a finite energy queue, a regular battery, and a single server. The size of the packet queue is denoted by , while the energy queue size is represented by . The regular battery has an infinite supply of energy, and each HP and LP packet requires one energy unit. The arrivals of one HP packet, one LP packet, two HP packets, two LP packet, and energy units are governed by Poisson processes, with respective arrival rates , , , and . The impatient time for each HP and LP packet waiting in the queue is determined by an exponential distribution, with corresponding rates and . The service time for HP and LP packets in the server is exponentially distributed, with associated rates and , respectively. Additionally, when the amount of harvested energy available in the energy queue is insufficient to support an HP or LP packet, the regular battery will be used based on probabilities and , respectively.

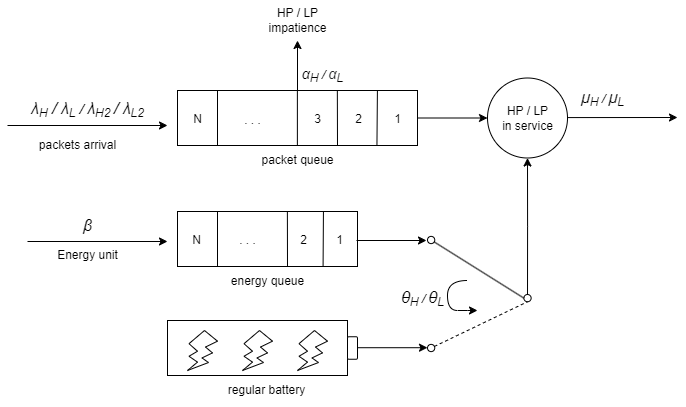


Fig. 3 - 1: The model diagram for scenario 1

### State balance equations

The system is modeled as a Markov chain with four dimensions: , where represents the number of high-priority (HP) packets in the system, represents the number of low-priority (LP) packets, represents the number of energy units in the energy queue, and represents the server status and the energy source used. The value of can take on five different values: (1) "" indicates that the server is idle; (2) "" indicates that an LP packet has entered the server and consumed one energy unit from the energy queue; (3) "" indicates that an HP packet has entered the server and consumed one energy unit from the energy queue; (4) "" indicates that an LP packet has entered the server and consumed one energy unit from the regular battery; (5) "" indicates that an HP packet has entered the server and consumed one energy unit from the regular battery. The steady state probability of the system is represented by , and the state space is defined as follows:

(3-1)

As a result, we can calculate the total count of possible states  
.

Based on the model description, there are 190 possible cases for the total system states. The balance equations for each of these states are presented below.

1. For , , , and ,
2. For , , , and ,
3. For , , , and ,
4. For , , , and ,
5. For , , , and ,
6. For , , , and ,
7. For , , , and ,
8. For , , , and ,
9. For , , , and ,
10. For , , , and ,
11. For , , , and ,
12. For , , , and ,
13. For , , , and ,
14. For , , , and ,
15. For , , , and ,
16. For , , , and ,
17. For , , , and ,
18. For , , , and ,
19. For , 3, , and ,
20. For , , , and ,
21. For , , , and ,
22. For , , , and ,
23. For , , , and ,
24. For , , , and ,
25. For 2, , , and ,
26. For , , , and ,
27. For , , , and ,
28. For 2, , , and ,
29. For , , and ,
30. For,,,and,
31. For,,,and,
32. For,,,and,
33. For,,,and,
34. For,,,and,
35. For,,,and,
36. For,,,and,
37. For,,,and,
38. For,,,and,
39. For,,,and,
40. For,,,and,
41. For,,,and,
42. For,,,and,
43. For,,,and,
44. For,,,and,
45. For,,,and,
46. For,,,and,
47. For,,,and,
48. For,,,and,
49. For,,,and,
50. For,,,and,
51. For,,,and,
52. For,,,and,
53. For,,,and,
54. For,,,and,
55. For,,,and,
56. For,,,and,
57. For,,,and,
58. For,,,and,
59. For,,,and,
60. For,,,and,
61. For,,,and,
62. For,,,and,
63. For,,,and,
64. For,,,and,
65. For,,,and,
66. For,,,and,
67. For , , , and ,
68. For , , , and ,
69. For , , , and ,
70. For , , , and ,
71. For , , and ,
72. For,,,and,
73. For,,,and,
74. For,,,and,
75. For,,,and,
76. For,,,and,
77. For,,,and,
78. For,,,and,
79. For,,,and,
80. For,,,and,
81. For,,,and,
82. For,,,and,
83. For,,,and,
84. For,,,and,
85. For,,,and,
86. For,,,and,
87. For,,,and,
88. For,,,and,
89. For,,,and,
90. For,,,and,
91. For,,,and,
92. For,,,and,
93. For,,,and,
94. For,,,and,
95. For,,,and,
96. For,,,and,
97. For,,,and,
98. For,,,and,
99. For,,,and,
100. For,,,and,
101. For,,,and,
102. For,,,and,
103. For,,,and,
104. For,,,and,
105. For,,,and,
106. For,,,and,
107. For,,,and,
108. For,,,and,
109. For,,,and,
110. For,,,and,
111. For,,,and,
112. For,,,and,
113. For,,,and,
114. For,,,and,
115. For,,,and,
116. For,,,and,
117. For,,,and,
118. For,,,and,
119. For,,,and,
120. For,,,and,
121. For,,,and,
122. For,,,and,
123. For,,,and,
124. For,,,and,
125. For,,,and,
126. For,,,and,
127. For,,,and,
128. For,,,and,
129. For,,,and,
130. For,,,and,
131. For,,,and,
132. For,,,and,
133. For,,,and,
134. For,,,and,
135. For,,,and,
136. For,,,and,
137. For,,,and,
138. For,,,and,
139. For,,,and,
140. For,,,and,
141. For,,,and,
142. For,,,and,
143. For,,,and,
144. For,,,and,
145. For,,,and,
146. For,,,and,
147. For,,,and,
148. For,,,and,
149. For,,,and,
150. For,,,and,
151. For,,,and,
152. For,,,and,
153. For,,,and,
154. For,,,and,
155. For,,,and,
156. For,,,and,
157. For,,,and,
158. For,,,and,
159. For,,,and,
160. For,,,and,
161. For,,,and,
162. For,,,and,
163. For,,,and,
164. For,,,and,
165. For,,,and,
166. For,,,and,
167. For,,,and,
168. For, ,,and,
169. For, ,,and,
170. For,,,and,
171. For,,,and,
172. For, ,,and,
173. For, ,,and,
174. For,,,and,
175. For,,,and,
176. For,,,and,
177. For,,,and,
178. For,,,and,
179. For,,,and,
180. For,,,and,
181. For,,,and,
182. For,,,and,

1. For,,,and,

1. For,,,and,

1. For,,,and,
2. For,,,and,

1. For,,,and,

1. For,,,and,

1. For,,,and,

1. For,,,and,

Since there are many equations presented above, discussing each one separately would be challenging. Therefore, we will focus on a relatively complicated case, specifically case 158, to provide an illustrative example. This state occurs when there are more than three but less than N-3 HP packets and more than two but less than N-i-1 LP packets in the system, and the packet queue is only one seat left, while the energy queue is empty. The HP packet being served in the server is using the regular battery. The corresponding detailed state transition diagram can be found in Fig. 3 - 2.

1. For,,,and,

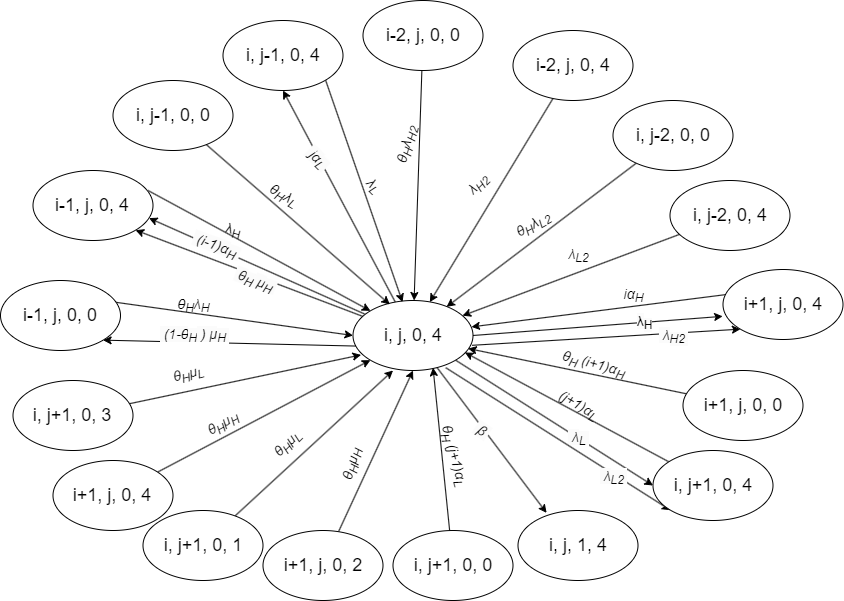


Fig. 3 - 2: The state transition diagram for ,,,and *.*

### Iterative algorithm

Using the iterative algorithm provided below, we perform calculations on the state balance equations until they converge, allowing us to determine the steady-state distribution of the system.

#### **Iterative algorithm:**

Step 1: Select a group of initial values for , , where is the total number of feasible states.

Step 2: Substitute into *Case 1* to *Case 190* to find , .

Step 3: Normalize , .

Step 4: If , stop the iterative algorithm, where is the stopping criterion. Otherwise, set , and return to Step 2.

In the analytical experiments, we set . It takes about 200 to 7000 iterations for the algorithm to converge.

### Performance measures

We obtained different performance measures of interest from the steady-state probability in order to evaluate the system's measures of effectiveness. These measures are presented below.

#### , the expected number of all packets in the system, is provided below.

(3-2)

(3-3)

(3-4)

#### , the expected number of all packets in the queue, is provided below.

(3-5)

(3-6)

(3-7)

#### , the throughput of all packets, is provided below.

(3-8)

(3-9)

(3-10)

#### , the blocking probability of each arrived packet, without considering its priority, is provided below.

(3-11)

#### , the energy loss probability, is provided below.

(3-12)

#### , the mean waiting time of all packets in the system, , which refers to all packets that have exited the system, either after receiving service or due to impatience, is provided below.

(3-13)

(3-14)

(3-15)

#### , the impatient loss probability of arrived packets, is provided below.

(3-16)

(3-17)

(3-18)

#### , the impatient loss probability of admitted packets, is provided below.

(3-19)

(3-20)

(3-21)

#### , the total loss probability of arrived packets, is provided below.

(3-22)

(3-23)

(3-24)

#### , the regular energy consumption ratio of all packets, is provided below.

(3-25)

(3-26)

(3-27)

## Scenario 2

In this section, we consider that all packets in this scenario can be classified into two priorities: high priority (HP) / low priority (LP), where the energy requirement of an HP packet is different from that of an LP packet. Packets with the same priority follow the FCFS discipline. Each packet entering the queue will be backlogged based on non-preemption, which means HP packets can always be sorted ahead of LP packets, but the LP packet in service cannot be interrupted and occupied. Besides, every packet waiting in the queue may randomly leave the system due to impatience. More importantly, we assume that whenever a packet is ready for service, it will first check whether there is a sufficient number of energy units in the energy queue, and if not, it will choose to use a regular battery based on a given probability instead. The associated model diagram, state balance equations, iterative algorithm, and performance measures are shown below.

### Model diagram

As shown in Fig. 3 - 3, the model in scenario 2 consists of: a finite packet queue, a finite energy queue, a regular battery, and a single server. It is assumed that the packet queue size is , , the energy queue size is , , and the regular battery has an unlimited supply of energy. Also, the energy requirement for each HP packet is defined as two energy units, and that of each LP packet as one energy unit. The arrivals of HP, LP packets and energy units follow a Poisson process with respective arrival rates , and . For each of HP and LP packets waiting in the queue, their impatient time is determined by an exponential distribution with corresponding rates and . The service time for each of HP and LP packets in the server is exponentially distributed with associated rates and , respectively. In addition, we assume that the regular battery will be used based on probabilities and , respectively, when the amount of harvested energy available in the energy queue is insufficient to support an HP or LP packet.



Fig. 3 - 3: The model diagram for scenario 2

### State balance equations

We describe the system as a four-dimensional Markov chain with the state , where denotes the number of HP packets in the system, denotes the number of LP packets in the system, denotes the number of harvested energy units in the energy queue, and denotes the server status and the energy resource being used. It is noted that there are five values which may take: (1) "" means the server is idle; (2) "" means that an LP packet enters the server by consuming one unit of energy from the energy queue; (3) "" means that an HP packet enters the server by consuming two units of energy from the energy queue; (4) "" means that an LP packet enters the server by consuming one unit of energy from the regular battery; (5) "" means that an HP packet enters the server by consuming two units of energy from the regular battery. The steady state probability of the system is denoted as . The state space is expressed as follows:

(3-28)

Thus, we can obtain the total number of feasible states  
*.*

According to the description of the model, the total system states can be divided into 135 cases. The corresponding state balance equations are shown as follows:

1. For , , , and ,
2. For , , , and ,
3. For , , , and ,
4. For , , , and ,
5. For , , , and ,
6. For , , , and ,
7. For , , , and ,
8. For , , , and ,
9. For , , , and ,
10. For , , , and ,
11. For , , , and ,
12. For , , , and ,
13. For , , , and ,
14. For , , , and ,
15. For , , , and ,
16. For , , , and ,
17. For , , , and ,
18. For , , , and ,
19. For , , , and ,
20. For , , , and ,
21. For , , , and ,
22. For , , , and ,
23. For , , , and ,
24. For , , , and ,
25. For , , , and ,
26. For , , , and ,
27. For , , , and ,
28. For,,,and,
29. For,,,and,
30. For,,,and,
31. For,,,and,
32. For,,,and,
33. For,,,and,
34. For,,,and,
35. For,,,and,
36. For,,,and,
37. For,,,and,
38. For,,,and,
39. For,,,and,
40. For,,,and,
41. For,,,and,
42. For,,,and,
43. For,,,and,
44. For,,,and,
45. For,,,and,
46. For,,,and,
47. For,,,and,
48. For,,,and,
49. For,,,and,
50. For,,,and,
51. For,,,and,
52. For,,,and,
53. For,,,and,
54. For,,,and,
55. For,,,and,
56. For,,,and,
57. For,,,and,
58. For,,,and,
59. For,,,and,
60. For,,,and,
61. For,,,and,
62. For,,,and,
63. For,,,and,
64. For,,,and,
65. For,,,and,
66. For,,,and,
67. For,,,and,
68. For,,,and,
69. For,,,and,
70. For,,,and,
71. For,,,and,
72. For,,,and,
73. For,,,and,
74. For,,,and,
75. For,,,and,
76. For,,,and,
77. For,,,and,
78. For,,,and,
79. For,,,and,
80. For,,,and,
81. For,,,and,
82. For,,,and,
83. For,,,and,
84. For,,,and,
85. For,,,and,
86. For,,,and,
87. For,,,and,
88. For,,,and,
89. For,,,and,
90. For,,,and,
91. For,,,and,
92. For,,,and,
93. For,,,and,
94. For,,,and,
95. For,,,and,
96. For,,,and,
97. For,,,and,
98. For,,,and,
99. For,,,and,
100. For,,,and,
101. For,,,and,
102. For,,,and,
103. For,,,and,
104. For,,,and,
105. For,,,and,
106. For,,,and,
107. For,,,and,
108. For,,,and,
109. For,,,and,
110. For,,,and,
111. For,,,and,
112. For,,,and,
113. For,,,and,
114. For,,,and,
115. For,,,and,
116. For,,,and,
117. For,,,and,
118. For, ,,and,
119. For,,,and,
120. For,,,and,
121. For,,,and,

1. For,,,and,

1. For,,,and,
2. For,,,and,
3. For,,,and,
4. For, ,,and,
5. For,,,and,
6. For,,,and,
7. For,,,and,
8. For,,,and,
9. For,,,and,
10. For,,,and,
11. For,,,and,

1. For,,,and,

1. For,,,and,

Considering the number of equations above, it would be difficult to discuss each one individually. Therefore, we take a relatively complex case, namely case 118, as an illustrative example. In this state, the system contains more than one but less than HP packets and at least one LP packet, and the packet queue is not full. The energy queue contains one unit of energy. The HP packet in the server is served by using the regular battery. The associated detailed state transition diagram is shown in Fig. 3 - 4.

For, ,,and,

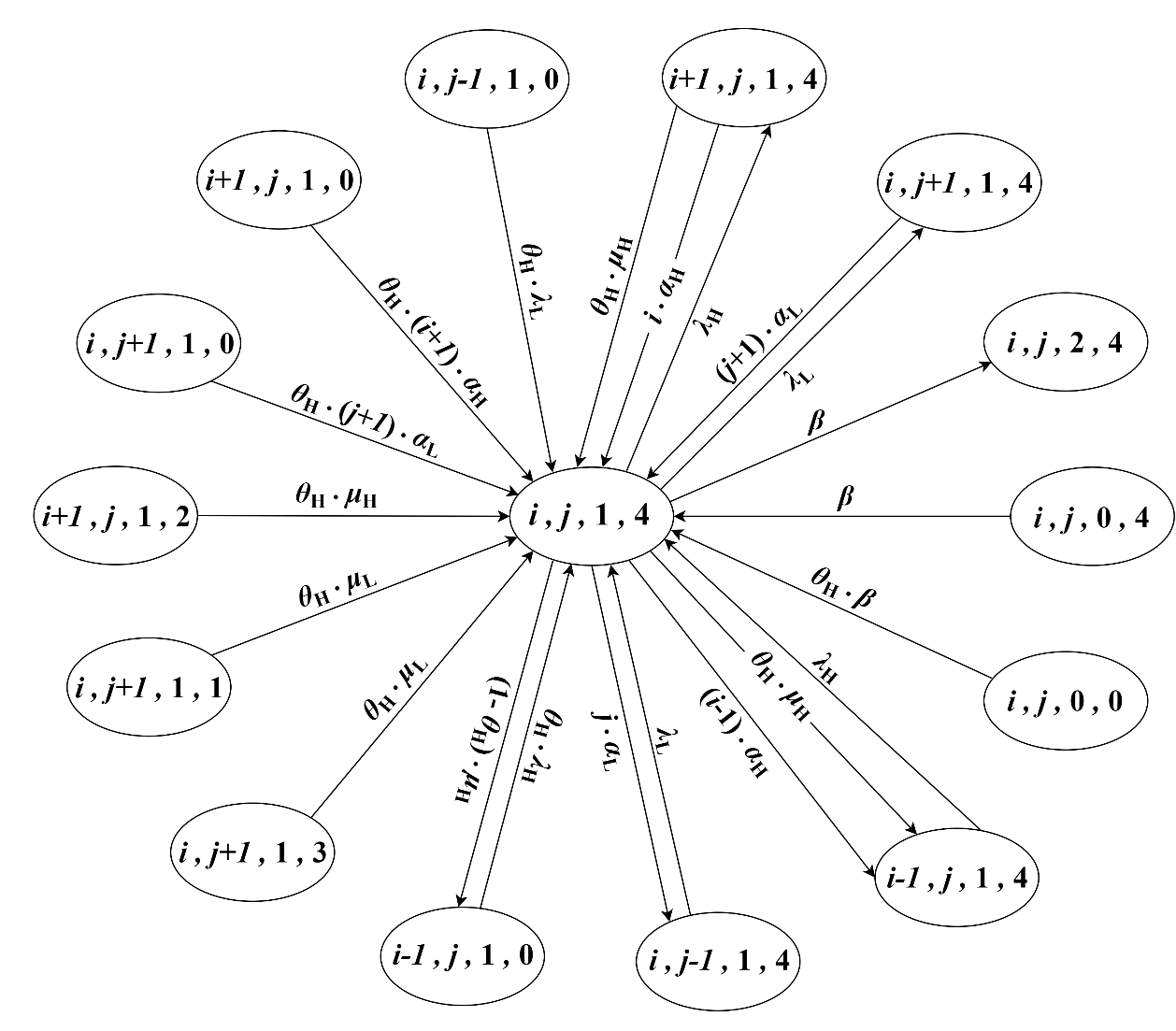


Fig. 3 - 4: The state transition diagram for ,,,and *.*

### Iterative algorithm

With the iterative algorithm shown below, we calculate the state balance equations until convergence is reached, and then obtain the steady-state distribution of the system.

#### **Iterative algorithm:**

Step 1: Choose a set of initial values for , , where is the total number of feasible states.

Step 2: Substitute into *Case 1* to *Case 135* to find , .

Step 3: Normalize , .

Step 4: If , stop the iterative algorithm, where is the stopping criterion. Otherwise, set , and return to Step 2.

In the analytical experiments, we set . The number of iterations required for the algorithm to converge is approximately 160 to 6000.

### Performance measures

In order to estimate the system's measures of effectiveness, we derived various performance measures of interest from the steady-state probability, which are shown as follows.

#### First, the expected number of all packets in the system, , is given below.

(3-29)

(3-30)

(3-31)

#### Second, the expected number of all packets in the queue, , is given below.

(3-32)

(3-33)

(3-34)

#### Third, the throughput of all packets, , is given below.

(3-35)

(3-36)

(3-37)

#### Fourth, the blocking probability of each arrived packet, , regardless of priority, is given below.

(3-38)

#### Fifth, the energy loss probability, , is given below.

(3-39)

#### Sixth, the mean waiting time of all packets in the system, , which includes the packets that have completed their service and those that have left the system due to impatience, is given below.

(3-40)

(3-41)

(3-42)

#### Seventh, the impatient loss probability of arrived packets, , is given below.

(3-43)

(3-44)

(3-45)

#### Eighth, the impatient loss probability of admitted packets, , is given below.

(3-46)

(3-47)

(3-48)

#### Ninth, the total loss probability of arrived packets, , is given below.

(3-49)

(3-50)

(3-51)

#### Tenth, the regular energy consumption ratio of all packets, , is given below.

(3-52)

(3-53)

(3-54)

## Scenario 3

In this section, we consider a three-connected-node network and all packets in this scenario can be classified into two priorities: high priority (HP) / low priority (LP), where both have the same energy requirements. It is defined that the first node is an "entry node", the second node is an "exit node", and the third node is a "control node". Once a packet passes through the control node, it will be assigned the correct path to the exit node and finally leave the network. Besides, every packet waiting in the queue may randomly leave the network due to impatience. More importantly, we assume that whenever a packet is ready for service at each node, it will first check whether there is a sufficient number of energy units in the energy queue, and if not, it will choose to use a regular battery based on a given probability instead. The associated model diagram, state balance equations, iterative algorithm, and performance measures are shown below.

### Model diagram

As shown in Fig. 3 - 5, there are three nodes connected into an open network model in scenario 3. It is assumed that each node consists of: a finite packet queue, a finite energy queue, a regular battery, and a single server. We define the packet queue size as , , the energy queue size as , , and the regular battery has an unlimited supply of energy. The energy requirement for both HP and LP packets is one energy unit. The arrivals of HP and LP packets from the outside network, as well as the arrival energy units of node follow a Poisson process with specific arrival rates , and . Furthermore, for each node in the network, we consider the sum of arrivals from external or internal HP (LP) packets as a Poisson process, and denote its arrival rates with (). The impatient time of each HP (LP) packet, which is waiting in the queue of node , is determined by an exponential distribution with corresponding rates (). The service time for each of HP (LP) packets in node is exponentially distributed with associated rates (). We assume that the regular battery will be used based on probabilities and , respectively, when the amount of harvested energy available in the energy queue is insufficient to support an HP or LP packet. Moreover, it should be noted that whenever a packet completes the service at node , it either leaves the network or is routed to the next node according to the assigned routing probability , where and . Nevertheless, the routing policy is subject to some restrictions: first, packets are not allowed to be routed from the exit node to the entry node; second, packets from the entry and exit nodes are only permitted to pass through the control node once, and then they will be forwarded to the previous node.



Fig. 3 - 5: The model diagram for scenario 3

### State balance equations

In order to simplify the derivation, each node in the network is assumed to be independent. We describe node as a four-dimensional Markov chain with the state , where denotes the number of HP packets in node , denotes the number of LP packets in node , denotes the number of harvested energy units in the energy queue at node , and denotes the server status and the energy resource being used in node . It is noted that there are five values which may take: (1) "" means the server is idle; (2) "" means that an LP packet enters the server by consuming one unit of energy from the energy queue; (3) "" means that an HP packet enters the server by consuming one unit of energy from the energy queue; (4) "" means that an LP packet enters the server by consuming one unit of energy from the regular battery; (5) "" means that an HP packet enters the server by consuming one unit of energy from the regular battery. The steady state probability of node is denoted as . The state space is expressed as follows:

(3-55)

Thus, we can obtain the total number of feasible states  
.

Moreover, there are five internal arrival rates that should be considered when analyzing the arrival rate of each node.

First, the HP and LP packet arrival rates from node 1 to node 3 are derived as

(3-56)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 1 that will be routed to node 3. The denominator part represents the total HP or LP packet arrival rate of node 1, which comprises those unblocked arrival rates from outside and node 3. The numerator part represents the unblocked arrival rate from outside that is routed to node 3 based on . It is noted that each packet can visit node 3 at most once, so only the external packets are allowed to be routed to node 3 in this case.

Second, the HP and LP packet arrival rates from node 3 to node 1 are derived as

(3-57)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 3 that will be routed to node 1. The denominator part represents the total HP or LP packet arrival rate of node 3, which comprises those unblocked arrival rates from node 1 and node 2. The numerator part represents the unblocked arrival rate from node 1. It is noted that each packet passing through the control node will be forwarded to the previous node. Therefore, the packets that have completed service at node 3 will be routed back to node 1 in this case.

Third, the HP and LP packet arrival rates from node 1 to node 2 are derived as

(3-58)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 1 that will be routed to node 2. The denominator part represents the total HP or LP packet arrival rate of node 1, which comprises those unblocked arrival rates from outside and node 3. The numerator part represents the unblocked arrival rates from outside that is routed to node 2 based on and the unblocked arrival rates from node 3. It is noted that each packet passing through the control node will be forwarded to the previous node. Therefore, the packets that have completed service at node 1 will be routed back to node 2 in this case.

Fourth, the HP and LP packet arrival rates from node 2 to node 3 are derived as

(3-59)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 2 that will be routed to node 3. The denominator part represents the total HP or LP packet arrival rate of node 2, which comprises those unblocked arrival rates from node 1 and node 3. The numerator part represents the unblocked arrival rate from node 1. Additionally, the second term indicates the portion of the HP or LP customers finishing service in node 1 that will be routed to node 3. The denominator part of the second term represents the HP or LP packet arrival rates being routed from node 1 to node 2, which comprises the arrival rates from node 3 to node 1 and the outside arrival rates that are routed to node 2 based on . The numerator part of the second term represents the outside packet arrival rates from node 1 routed to node 2 based on and then routed to node 3 based on .

Fifth, the HP and LP packet arrival rates from node 3 to node 2 are derived as

(3-60)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 3 that will be routed to node 2. The denominator part represents the total HP or LP packet arrival rate of node 3, which comprises those unblocked arrival rates from node 1 and node 2. The numerator part represents the unblocked arrival rate from node 2. It is noted that each packet passing through the control node will be forwarded to the previous node. Therefore, the packets that have completed service at node 3 will be routed back to node 2 in this case.

According to the above description of the model, the HP and LP packet arrival rates for each node , and , are given by

, (3-61)

In addition, the system states for each node can be divided into 101 cases. The corresponding state balance equations are shown as follows:

1. For , , , and ,
2. For , , , and ,
3. For , , , and ,
4. For , , , and ,
5. For , , , and ,
6. For , , , and ,
7. For , , , and ,
8. For , , , and ,
9. For , , , and ,
10. For , , , and ,
11. For , , , and ,
12. For , , , and ,
13. For , , , and ,
14. For , , , and ,
15. For , , , and ,
16. For , , , and ,
17. For , , , and ,
18. For , , , and ,
19. For,,,and,
20. For,,,and,
21. For,,,and,
22. For,,,and,
23. For,,,and,
24. For,,,and,
25. For,,,and,
26. For,,,and,
27. For,,,and,
28. For,,,and,
29. For,,,and,
30. For,,,and,
31. For,,,and,
32. For,,,and,
33. For,,,and,
34. For,,,and,
35. For,,,and,
36. For,,,and,
37. For,,,and,
38. For,,,and,
39. For,,,and,
40. For,,,and,
41. For,,,and,
42. For,,,and,
43. For,,,and,
44. For,,,and,
45. For,,,and,
46. For,,,and,
47. For,,,and,
48. For,,,and,
49. For,,,and,
50. For,,,and,
51. For,,,and,
52. For,,,and,
53. For,,,and,
54. For,,,and,
55. For,,,and,
56. For,,,and,
57. For,,,and,
58. For,,,and,
59. For,,,and,
60. For,,,and,
61. For,,,and,
62. For,,,and,
63. For,,,and,
64. For,,,and,
65. For,,,and,
66. For,,,and,
67. For,,,and,
68. For,,,and,
69. For,,,and,
70. For,,,and,
71. For,,,and,
72. For,,,and,
73. For,,,and,
74. For,,,and,
75. For,,,and,
76. For,,,and,
77. For,,,and,
78. For,,,and,
79. For,,,and,
80. For,,,and,
81. For,,,and,
82. For,,,and,
83. For,,,and,
84. For,,,and,
85. For,,,and,
86. For,,,and,
87. For,,,and,
88. For,,,and,
89. For,,,and,
90. For,,,and,
91. For,,,and,
92. For, ,,and,
93. For,,,and,
94. For,,,and,
95. For,,,and,
96. For,,,and,
97. For,,,and,
98. For,,,and,
99. For,,,and,

1. For,,,and,

1. For,,,and,

### Iterative algorithm

With the iterative algorithm shown below, we calculate the state balance equations until convergence is reached, and then obtain the steady-state distribution of the network.

#### **Iterative algorithm:**

Step 1: Choose a set of initial values for , , where is the total number of feasible states for node , .

Step 2: Calculate , , and based on , .

Step 3: Substitute , , and into eqs. (3-56) - (3-61) to find and , .

Step 4: Substitute into *Case 1* to *Case 101* to find   
, , .

Step 5: Normalize , , .

Step 6: If , stop the iterative algorithm, where is the stopping criterion. Otherwise, set , and return to Step 2.

In the analytical experiments, we set . The number of iterations required for the algorithm to converge is approximately 250 to 7500.

### Performance measures

In order to estimate the network's measures of effectiveness, we derived various performance measures of interest from the steady-state probability of each node , which are shown as follows.

#### First, the expected number of all packets in node , (, is given below.

(3-62)

(3-63)

(3-64)

#### Second, the expected number of all packets for the network, , is given below.

(3-65)

(3-66)

(3-67)

#### Third, the expected number of all packets in the queue of node , , is given below.

(3-68)

(3-69)

(3-70)

#### Fourth, the expected number of all packets in the queue, , is given below.

(3-71)

(3-72)

(3-73)

#### Fifth, the throughput of all packets for node , , is given below.

(3-74)

(3-75)

(3-76)

#### Sixth, the throughput of all packets for the network, , is given below.

(3-77)

(3-78)

(3-79)

#### Seventh, the blocking probability of each arrived packet for node , , regardless of priority, is given below.

(3-80)

#### Eighth, the blocking probability of each arrived packet for the network, , regardless of priority, is given below.

(3-81)

#### Ninth, the energy loss probability for node , , is given below.

(3-82)

#### Tenth, the energy loss probability for the network, , is given below.

(3-83)

#### Eleventh, the mean waiting time of all packets in node , , which includes the packets that have completed their service and those that have left the network due to impatience, is given below.

(3-84)

(3-85)

(3-86)

#### Twelfth, the mean waiting time of all packets in the network, , which includes the packets that have completed their service and those that have left the network due to impatience, is given below.

(3-87)

(3-88)

(3-89)

#### Thirteenth, the impatient loss probability of arrived packets for node , , is given below.

(3-90)

(3-91)

(3-92)

#### Fourteenth, the impatient loss probability of arrived packets for the network, , is given below.

(3-93)

(3-94)

(3-95)

#### Fifteenth, the impatient loss probability of admitted packets for node , , is given below.

(3-96)

(3-97)

(3-98)

#### Sixteenth, the impatient loss probability of admitted packets for the network, , is given below.

(3-99)

(3-100)

(3-101)

#### Seventeenth, the total loss probability of arrived packets for node , , is given below.

(3-102)

(3-103)

(3-104)

#### Eighteenth, the total loss probability of arrived packets for the network, , is given below.

(3-105)

(3-106)

(3-107)

#### Nineteenth, the regular energy consumption ratio of all packets for node , , is given below.

(3-108)

(3-109)

(3-110)

#### Twentieth, the regular energy consumption ratio of all packets for the network, , is given below.

(3-111)

(3-112)

(3-113)

## Scenario 4

In this section, we consider a three-connected-node network and all packets in this scenario can be classified into two priorities: high priority (HP) / low priority (LP), where the energy requirement of an HP packet is different from that of an LP packet. It is defined that the first node is an "entry node", the second node is an "exit node", and the third node is a "control node". Once a packet passes through the control node, it will be assigned the correct path to the exit node and finally leave the network. Besides, every packet waiting in the queue may randomly leave the network due to impatience. More importantly, we assume that whenever a packet is ready for service at each node, it will first check whether there is a sufficient number of energy units in the energy queue, and if not, it will choose to use a regular battery based on a given probability instead. The associated model diagram, state balance equations, iterative algorithm, and performance measures are shown below.

### Model diagram

As shown in Fig. 3 - 6, there are three nodes connected into an open network model in scenario 4. It is assumed that each node consists of: a finite packet queue, a finite energy queue, a regular battery, and a single server. We define the packet queue size as , , the energy queue size as , , and the regular battery has an unlimited supply of energy. The energy requirement for each HP packet is defined as two energy units, and that of each LP packet as one energy unit. The arrivals of HP and LP packets from the outside network, as well as the arrival energy units of node follow a Poisson process with specific arrival rates , and . Furthermore, for each node in the network, we consider the sum of arrivals from external or internal HP (LP) packets as a Poisson process, and denote its arrival rates with (). The impatient time of each HP (LP) packet, which is waiting in the queue of node , is determined by an exponential distribution with corresponding rates (). The service time for each of HP (LP) packets in node is exponentially distributed with associated rates (). We assume that the regular battery will be used based on probabilities and , respectively, when the amount of harvested energy available in the energy queue is insufficient to support an HP or LP packet. Moreover, it should be noted that whenever a packet completes the service at node , it either leaves the network or is routed to the next node according to the assigned routing probability , where and . Nevertheless, the routing policy is subject to some restrictions: first, packets are not allowed to be routed from the exit node to the entry node; second, packets from the entry and exit nodes are only permitted to pass through the control node once, and then they will be forwarded to the previous node.



Fig. 3 - 6: The model diagram for scenario 4

### State balance equations

In order to simplify the derivation, each node in the network is assumed to be independent. We describe node as a four-dimensional Markov chain with the state , where denotes the number of HP packets in node , denotes the number of LP packets in node , denotes the number of harvested energy units in the energy queue at node , and denotes the server status and the energy resource being used in node . It is noted that there are five values which may take: (1) "" means the server is idle; (2) "" means that an LP packet enters the server by consuming one unit of energy from the energy queue; (3) "" means that an HP packet enters the server by consuming two units of energy from the energy queue; (4) "" means that an LP packet enters the server by consuming one unit of energy from the regular battery; (5) "" means that an HP packet enters the server by consuming two units of energy from the regular battery. The steady state probability of node is denoted as . The state space is expressed as follows:

(3-114)

Thus, we can obtain the total number of feasible states  
.

Moreover, there are five internal arrival rates that should be considered when analyzing the arrival rate of each node.

First, the HP and LP packet arrival rates from node 1 to node 3 are derived as

(3-115)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 1 that will be routed to node 3. The denominator part represents the total HP or LP packet arrival rate of node 1, which comprises those unblocked arrival rates from outside and node 3. The numerator part represents the unblocked arrival rate from outside that is routed to node 3 based on . It is noted that each packet can visit node 3 at most once, so only the external packets are allowed to be routed to node 3 in this case.

Second, the HP and LP packet arrival rates from node 3 to node 1 are derived as

(3-116)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 3 that will be routed to node 1. The denominator part represents the total HP or LP packet arrival rate of node 3, which comprises those unblocked arrival rates from node 1 and node 2. The numerator part represents the unblocked arrival rate from node 1. It is noted that each packet passing through the control node will be forwarded to the previous node. Therefore, the packets that have completed service at node 3 will be routed back to node 1 in this case.

Third, the HP and LP packet arrival rates from node 1 to node 2 are derived as

(3-117)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 1 that will be routed to node 2. The denominator part represents the total HP or LP packet arrival rate of node 1, which comprises those unblocked arrival rates from outside and node 3. The numerator part represents the unblocked arrival rates from outside that is routed to node 2 based on and the unblocked arrival rates from node 3. It is noted that each packet passing through the control node will be forwarded to the previous node. Therefore, the packets that have completed service at node 1 will be routed back to node 2 in this case.

Fourth, the HP and LP packet arrival rates from node 2 to node 3 are derived as

(3-118)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 2 that will be routed to node 3. The denominator part represents the total HP or LP packet arrival rate of node 2, which comprises those unblocked arrival rates from node 1 and node 3. The numerator part represents the unblocked arrival rate from node 1. Additionally, the second term indicates the portion of the HP or LP customers finishing service in node 1 that will be routed to node 3. The denominator part of the second term represents the HP or LP packet arrival rates being routed from node 1 to node 2, which comprises the arrival rates from node 3 to node 1 and the outside arrival rates that are routed to node 2 based on . The numerator part of the second term represents the outside packet arrival rates from node 1 routed to node 2 based on and then routed to node 3 based on .

Fifth, the HP and LP packet arrival rates from node 3 to node 2 are derived as

(3-119)

For the right-hand side of this equation, the first term indicates the proportion of HP or LP packets finishing service in node 3 that will be routed to node 2. The denominator part represents the total HP or LP packet arrival rate of node 3, which comprises those unblocked arrival rates from node 1 and node 2. The numerator part represents the unblocked arrival rate from node 2. It is noted that each packet passing through the control node will be forwarded to the previous node. Therefore, the packets that have completed service at node 3 will be routed back to node 2 in this case.

According to the above description of the model, the HP and LP packet arrival rates for each node , and , are given by

, (3-120)

In addition, the system states for each node can be divided into 135 cases. The corresponding state balance equations are shown as follows:

1. For , , , and ,
2. For , , , and ,
3. For , , , and ,
4. For , , , and ,
5. For , , , and ,
6. For , , , and ,
7. For , , , and ,
8. For , , , and ,
9. For , , , and ,
10. For , , , and ,
11. For , , , and ,
12. For , , , and ,
13. For , , , and ,
14. For , , , and ,
15. For , , , and ,
16. For , , , and ,
17. For , , , and ,
18. For , , , and ,
19. For , , , and ,
20. For , , , and ,
21. For , , , and ,
22. For , , , and ,
23. For , , , and ,
24. For , , , and ,
25. For , , , and ,
26. For , , , and ,
27. For , , , and ,
28. For,,,and,
29. For,,,and,
30. For,,,and,
31. For,,,and,
32. For,,,and,
33. For,,,and,
34. For,,,and,
35. For,,,and,
36. For,,,and,
37. For,,,and,
38. For,,,and,
39. For,,,and,
40. For,,,and,
41. For,,,and,
42. For,,,and,
43. For,,,and,
44. For,,,and,
45. For,,,and,
46. For,,,and,
47. For,,,and,
48. For,,,and,
49. For,,,and,
50. For,,,and,
51. For,,,and,
52. For,,,and,
53. For,,,and,
54. For,,,and,
55. For,,,and,
56. For,,,and,
57. For,,,and,
58. For,,,and,
59. For,,,and,
60. For,,,and,
61. For,,,and,
62. For,,,and,
63. For,,,and,
64. For,,,and,
65. For,,,and,
66. For,,,and,
67. For,,,and,
68. For,,,and,
69. For,,,and,
70. For,,,and,
71. For,,,and,
72. For,,,and,
73. For,,,and,
74. For,,,and,
75. For,,,and,
76. For,,,and,
77. For,,,and,
78. For,,,and,
79. For,,,and,
80. For,,,and,
81. For,,,and,
82. For,,,and,
83. For,,,and,
84. For,,,and,
85. For,,,and,
86. For,,,and,
87. For,,,and,
88. For,,,and,
89. For,,,and,
90. For,,,and,
91. For,,,and,
92. For,,,and,
93. For,,,and,
94. For,,,and,
95. For,,,and,
96. For,,,and,
97. For,,,and,
98. For,,,and,
99. For,,,and,
100. For,,,and,
101. For,,,and,
102. For,,,and,
103. For,,,and,
104. For,,,and,
105. For,,,and,
106. For,,,and,
107. For,,,and,
108. For,,,and,
109. For,,,and,
110. For,,,and,
111. For,,,and,
112. For,,,and,
113. For,,,and,
114. For,,,and,
115. For,,,and,
116. For,,,and,
117. For,,,and,
118. For, ,,and,
119. For,,,and,
120. For,,,and,
121. For,,,and,

1. For,,,and,

1. For,,,and,
2. For,,,and,
3. For,,,and,
4. For, ,,and,
5. For,,,and,
6. For,,,and,
7. For,,,and,
8. For,,,and,
9. For,,,and,
10. For,,,and,
11. For,,,and,

1. For,,,and,

1. For,,,and,

### Iterative algorithm

With the iterative algorithm shown below, we calculate the state balance equations until convergence is reached, and then obtain the steady-state distribution of the network.

#### **Iterative algorithm:**

Step 1: Choose a set of initial values for , , where is the total number of feasible states for node , .

Step 2: Calculate , , and based on , .

Step 3: Substitute , , and into eqs. (3-115) - (3-120) to find and , .

Step 4: Substitute into *Case 1* to *Case 135* to find   
, , .

Step 5: Normalize , , .

Step 6: If , stop the iterative algorithm, where is the stopping criterion. Otherwise, set , and return to Step 2.

In the analytical experiments, we set . The number of iterations required for the algorithm to converge is approximately 200 to 4200.

### Performance measures

In order to estimate the network's measures of effectiveness, we derived various performance measures of interest from the steady-state probability of each node , which are shown as follows.

#### First, the expected number of all packets in node , (, is given below.

(3-121)

(3-122)

(3-123)

#### Second, the expected number of all packets for the network, , is given below.

(3-124)

(3-125)

(3-126)

#### Third, the expected number of all packets in the queue of node , , is given below.

(3-127)

(3-128)

(3-129)

#### Fourth, the expected number of all packets in the queue, , is given below.

(3-130)

(3-131)

(3-132)

#### Fifth, the throughput of all packets for node , , is given below.

(3-133)

(3-134)

(3-135)

#### Sixth, the throughput of all packets for the network, , is given below.

(3-136)

(3-137)

(3-138)

#### Seventh, the blocking probability of each arrived packet for node , , regardless of priority, is given below.

(3-139)

#### Eighth, the blocking probability of each arrived packet for the network, , regardless of priority, is given below.

(3-140)

#### Ninth, the energy loss probability for node , , is given below.

(3-141)

#### Tenth, the energy loss probability for the network, , is given below.

(3-142)

#### Eleventh, the mean waiting time of all packets in node , , which includes the packets that have completed their service and those that have left the network due to impatience, is given below.

(3-143)

(3-144)

(3-145)

#### Twelfth, the mean waiting time of all packets in the network, , which includes the packets that have completed their service and those that have left the network due to impatience, is given below.

(3-146)

(3-147)

(3-148)

#### Thirteenth, the impatient loss probability of arrived packets for node , , is given below.

(3-149)

(3-150)

(3-151)

#### Fourteenth, the impatient loss probability of arrived packets for the network, , is given below.

(3-152)

(3-153)

(3-154)

#### Fifteenth, the impatient loss probability of admitted packets for node , , is given below.

(3-155)

(3-156)

(3-157)

#### Sixteenth, the impatient loss probability of admitted packets for the network, , is given below.

(3-158)

(3-159)

(3-160)

#### Seventeenth, the total loss probability of arrived packets for node , , is given below.

(3-161)

(3-162)

(3-163)

#### Eighteenth, the total loss probability of arrived packets for the network, , is given below.

(3-164)

(3-165)

(3-166)

#### Nineteenth, the regular energy consumption ratio of all packets for node , , is given below.

(3-167)

(3-168)

(3-169)

#### Twentieth, the regular energy consumption ratio of all packets for the network, , is given below.

(3-170)

(3-171)

(3-172)